Chen, Zhiyong. “A cooperative target-fencing protocol of multiple vehicles”. Published in *Automatica* Vol. 107, p. 591-594 (2019).

Available from: http://dx.doi.org/10.1016/j.automatica.2019.05.034

A Cooperative Target-Fencing Protocol of Multiple Vehicles

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Abstract

A class of cooperative controllers is designed for a group of autonomous vehicles, using the relative positions between geographical neighbors and a specified target. The controller is equipped with three components: attractive component that drives each vehicle towards the target, repulsive component between adjacent vehicles, and rotation component for neighbored vehicles aligned with the target in a straight line. It is proved that the vehicles with the proposed autonomous controller can asymptotically fence a specified target to their convex hull. Meanwhile, the vehicles do not collide and they are not stuck in a singleton formation.

Key words: Autonomous vehicles, Networks, Multi-agent systems, Formation control, Collision avoidance

1 Introduction

The cooperative target-fencing problem studied in this paper aims at a class of autonomous controllers that can drive a group of vehicles to asymptotically fence a specified target to their convex hull. It is technically relevant to various cooperative formation control problems, for example, the circular motion in [1, 3, 10]. These circular formation control formulations do not essentially involve a physical target. The other class of relevant research was formulated as a target-enclosing or target-capturing problem. For instance, a cyclic pursuit strategy was used in [7] to drive agents to enclose the target object, called target-capturing. The technique of enclosing a target by holonomic or nonholonomic mobile vehicles was reported in [5, 11], where vehicles eventually move on a circle centered at the target with a predefined stand-off distance.

In the aforementioned results, the group of agents aim to enclose a target within their *moving trajectories*. The agents do not necessarily enclose a target at every moment, even when the desired behavior is achieved. However, the target-fencing protocol studied in this paper aims to drive a group of vehicles to asymptotically approach some desired trajectories that fence a specified target to their *convex hull*, at every moment. A dual problem of target-fencing is the so-called containment control studied in, e.g., [6], that aims to drive a group of agents to be contained in an area fenced by another group of targets. The target-fencing problem is also called surrounding control in, e.g., [2, 8]. A control approach was proposed in [2] by assuming that the vehicles are initially placed within a circle and/or using a predefined stand-off distance between the vehicles and the target. In [8], the surrounding formation is specified by a complex-value adjacency matrix. A predefined distance was also used in the aforementioned target-enclosing or target-capturing problem such as [5, 11].

The main objective of this paper is to develop a novel class of cooperative target-fencing controllers for a group of autonomous vehicles, without a specified stand-off distance or formation. In other words, the distance between the vehicles and the target is autonomously maintained. Each controller uses only the relative positions between geographical neighbors and the target. The novelty of the controller is that it is equipped with three fundamental functionalities: attractive component that drives each vehicle towards the target, repulsive component between adjacent vehicles, and rotation component for neighbored vehicles aligned with the target in a straight line. It is proved that the group of vehicles with the controller can asymptotically fence a specified target to their convex hull. Meanwhile, the vehicles do not collide and they are not stuck in a singleton formation. It is worth mentioning that collision avoidance has its independent theoretical interest and practical importance and it has been extensively studied for various scenarios in, e.g., [4, 9]. It is novel to include collision avoidance in the proposed target-fencing scenario.

⋆ This paper was not presented at any IFAC meeting. The work was substantially supported by the National Natural Science Foundation of China under grant No. 51729501.

2 Main Results

Let $\mathbb{N} = \{1, 2, \cdots, N\}$. Denote the complete position distribution of a multi-agent system by $x = [x_1^\intercal, \ldots, x_N^\intercal]^\intercal$, where $x_i = [x_{1,i}, x_{2,i}]^\intercal \in \mathbb{R}^2$, $i \in \mathbb{N}$, represents the Cartesian coordinates of the i-th vehicle. The kinematics model of each vehicle is represented by a continuous-time equation

$$\dot{x}_i(t) = u_i(t),\ i \in \mathbb{N} \tag{1}$$

with the input u_i to be designed. Throughout the paper, the time argument (t) is ignored for neatness when no confusion is caused. Denote the convex hull of $x_1, \cdots, x_N$ by $\mathrm{co}(x)$, that is,

$$\mathrm{co}(x) = \left\{ \sum_{i=1}^{N} \lambda_i x_i :\ \lambda_i \geq 0, \forall i \text{ and} \sum_{i=1}^{N} \lambda_i = 1 \right\}.$$

Let $P_{x_o}(x) = \min_{s \in \mathrm{co}(x)} \|x_o - s\|$ be the distance between a point x_o and $\mathrm{co}(x)$ and $p_{x_o}(x) = \arg\min_{s \in \mathrm{co}(x)} \|x_o - s\|$ be the point in $\mathrm{co}(x)$ that has the distance from x_o. Obviously, $x_o \in \mathrm{co}(x)$ if and only if $P_{x_o}(x) = 0$. Let $\mathrm{area}(x)$ be the area of $\mathrm{co}(x)$, i.e., the Lebesgue measure of the subset $\mathrm{co}(x)$ of the two-dimensional Euclidean space. The vehicles form a straight line when $\mathrm{area}(x) = 0$. Throughout the paper, it is assumed that the number of vehicles $N \geq 3$ such that $\mathrm{area}(x) \neq 0$ for some position distribution x. For a specified target position $x_o \in \mathbb{R}^2$, define the augmented position distribution $x^a = [x_o^\intercal, x^\intercal]^\intercal$. Similarly, we denote the convex hull of $x_o, x_1, \cdots, x_N$ by $\mathrm{co}(x^a)$ and the area of $\mathrm{co}(x^a)$ by $\mathrm{area}(x^a)$.

The target-fencing problem aims to design the controller u_i of the model (1), for a specified target position $x_o \in \mathbb{R}^2$, such that the closed-loop system satisfies the following properties

- *(P1) the target x_o is asymptotically fenced by the vehicles in the sense of $\lim_{t\to\infty} P_{x_o}(x(t)) = 0$;*
- *(P2) collision among vehicles is avoided in the sense of $\|x_i(t) - x_j(t)\| > d,\ \forall t \geq 0, i \neq j \in \mathbb{N}$ for a specified $d > 0$;*
- *(P3) the set*

$$\mathcal{S} = \{x \in \mathbb{R}^{2N} \mid \mathrm{area}(x^a) = 0, \|x_i - x_j\| > d, i \neq j \in \mathbb{N}\},$$

called a singleton formation (a straight line), is not an invariant set.

Remark 2.1 The property (P1) has a two-fold meaning. One one hand, there exist some desired trajectories, represented by

$$x^*(t) = x(t) - \mathbf{1} \otimes [p_{x_o}(x(t)) - x_o],$$

where $\mathbf{1}$ is a N-dimensional vector of elements 1 and $\otimes$ represents the Kronecker product. Due to the following fact

$$\begin{aligned} P_{x_o}(x^*(t)) &= \min_{s \in \mathrm{co}(x^*(t))} \|x_o - s\| \\ &= \min_{s \in \mathrm{co}(x(t))} \|x_o - (s - (p_{x_o}(x(t)) - x_o))\| \\ &= \min_{s \in \mathrm{co}(x(t))} \|s - p_{x_o}(x(t))\| = 0,\ \forall t \end{aligned}$$

the desired trajectories fence the target to their convex hull, at every moment. One the other hand, the vehicles asymptotically approach the desired trajectories as

$$\begin{aligned} \lim_{t\to\infty} \|x(t) - x^*(t)\| &= \sqrt{N} \lim_{t\to\infty} \|p_{x_o}(x(t)) - x_o\| \\ &= \sqrt{N} \lim_{t\to\infty} P_{x_o}(x(t)) = 0. \end{aligned}$$

Remark 2.2 The property (P2) excludes collision among vehicles. However, it should be noted that collision between a vehicle and the target is not an issue considered in this framework. This scenario can find applications such as roundup of a ground target by flying drones where we must consider collision avoidance among drones but the ground target and a flying drone never collide at different heights. Meanwhile, the control strategy can be developed on a two-dimensional plane as formulated in this paper without considering the vertical dimension.

Remark 2.3 The property (P3) guarantees that the vehicles do not stay in the singleton formation $\mathcal{S}$ forever. Theoretically, they may reach the singleton formation $\mathcal{S}$ periodically, but will move out of $\mathcal{S}$ when it occurs.

Define the set of geographical neighbors of the i-th vehicle as

$$\mathcal{N}_i(t) = \{j \in \mathbb{N} : j \neq i \,|\, \|x_i(t) - x_j(t)\| \leq \mu\},\ i \in \mathbb{N}$$

with a specified vision distance $\mu > d > 0$ where d is the threshold of collision avoidance. As the neighborhood is symmetrically defined according to the distance of two agents, the underlying graph is thus undirected. Consider the control input $u_i = u_i^o$ for each vehicle, with $x_{ij} := x_i - x_j$ and u_i^o defined as follow

$$\begin{aligned} u_i^o = &\sum_{j \in \mathcal{N}_i} \alpha(\|x_{ij}\|) \frac{x_{ij}}{\|x_{ij}\|} + \sum_{j \in \mathcal{N}_i} \beta(x_i, x_j, x_o) R \frac{x_{ij}}{\|x_{ij}\|} \\ &+ k(x_o - x_i),\ i \in \mathbb{N}, \end{aligned} \tag{2}$$

where $k > 0$ is a constant gain and R is the $90°$ counter clockwise rotation matrix. The function $\alpha : (d, \infty) \mapsto [0, \infty)$ is continuous and satisfies $\alpha(s) = 0, \forall s \in [\mu, \infty)$ and $\lim_{s\to d} \alpha(s) = \infty$. The function β is explicitly given by $\beta(x_i, x_j, x_o) = \epsilon \max\{0, \delta - \angle(x_{io}, x_{jo})\}$ for two constants $\delta \in [0, \pi/2)$ and $\epsilon > 0$, where

$$\angle(x_{io}, x_{jo}) = \arccos\left(\frac{x_{io} \cdot x_{jo}}{\|x_{io}\| \|x_{jo}\|}\right) \in [0, \pi]$$

is the angle of the two vectors x_{io} and x_{jo}. Here, $\boldsymbol{\cdot}$ is the dot product operator. Let $\angle(0,a) = \angle(a,0) = \pi$ for completeness of notation.

The controller (2) contains three fundamental functionalities. The attractive component $k(x_o - x_i)$ drives each vehicle towards the target. The repulsive component $\sum_{j\in\mathcal{N}_i} \alpha(\|x_{ij}\|)\frac{x_{ij}}{\|x_{ij}\|}$ between two geographically neighbored vehicles works to avoid their collision. These two components eventually reach balance when the vehicles approach appropriate positions that fence the target within their convex hull. The additional component $\sum_{j\in\mathcal{N}_i} \beta(x_i, x_j, x_o)Rx_{ij}/\|x_{ij}\|$ takes effect when two vehicles and the target are almost aligned in a straight line. In particular, the rotation due to the matrix R drives an agent to turn and thus escape the singleton formation, when the angle of the two vectors x_{io} and x_{jo} is smaller than the specified parameter δ in the function β. The other parameter ϵ in β represents the strength of the rotation component.

Theorem 2.1 *For the system (1) with the control input $u_i = u_i^o$ defined in (2), the target-fencing problem is solved in the sense of (P1), (P2), and (P3) for $N > 3$, provided that the vehicles do not initially collide, i.e., $\|x_i(0) - x_j(0)\| > d, i \neq j \in \mathbb{N}$. Moreover, the result still holds for $u_i = u_i^o + e_i$, subject to any input disturbance satisfying $\lim_{t\to\infty} e_i(t) = 0,\ i \in \mathbb{N}$, exponentially.*

Proof: Let $\bar{x} = \sum_{i\in\mathbb{N}} x_i/N$ be the center of the vehicles and $\bar{e} = \sum_{i\in\mathbb{N}} e_i/N$ be the average of input disturbances. A direct calculation shows

$$\begin{aligned}\dot{\bar{x}} =& \frac{1}{N}\sum_{i\in\mathbb{N}}\sum_{j\in\mathcal{N}_i} \alpha(\|x_{ij}\|)\frac{x_{ij}}{\|x_{ij}\|} \\ &+ \frac{1}{N}\sum_{i\in\mathbb{N}}\sum_{j\in\mathcal{N}_i} \beta(x_i, x_j, x_o)R\frac{x_{ij}}{\|x_{ij}\|} \\ &+ \frac{1}{N}\sum_{i\in\mathbb{N}} k(x_o - x_i) + \frac{1}{N}\sum_{i\in\mathbb{N}} e_i \\ =& -k\bar{x} + kx_o + \bar{e},\end{aligned}$$

due to the facts $i \in \mathcal{N}_j \Leftrightarrow j \in \mathcal{N}_i$, $x_{ij} = -x_{ji}$, and $\beta(x_i, x_j, x_o) = \beta(x_j, x_i, x_o)$. For this linear system, it is obvious to see that $\lim_{t\to\infty} \bar{x}(t) - x_o = 0$ as $\lim_{t\to\infty} \bar{e}(t) = 0$ exponentially. It, together with the fact $\bar{x} \in \mathrm{co}(x)$, further implies $\lim_{t\to\infty} P_{x_o}(x(t)) = 0$. The property (P1) is proved.

To prove the property (P2), we define a potential energy function

$$V(x) = \frac{1}{2}\sum_{i\in\mathbb{N}}\sum_{j\in\mathcal{N}_i}\int_{\|x_{ij}\|}^{\mu} \alpha(s)ds + \frac{k}{2}\sum_{i\in\mathbb{N}}\|x_i - x_o\|^2. \tag{3}$$

Obviously, $V(x)$ is nonnegative. We will next show the change of $V(x)$ along the trajectory of the closed-loop system. Denote

$$\phi_i = \sum_{j\in\mathcal{N}_i} \alpha(\|x_{ij}\|)\frac{x_{ij}}{\|x_{ij}\|},\ \psi_i = \sum_{j\in\mathcal{N}_i} \beta(x_i, x_j, x_o)R\frac{x_{ij}}{\|x_{ij}\|}.$$

One has $\|\psi_i\| \leq (N-1)\epsilon\delta$. It is noted that

$$\begin{aligned}\frac{\partial V(x)}{\partial x_i} &= -\sum_{j\in\mathcal{N}_i} \alpha(\|x_{ij}\|)\frac{x_{ij}^{\mathsf{T}}}{\|x_{ij}\|} + k(x_i - x_o)^{\mathsf{T}} \\ &= -\phi_i^{\mathsf{T}} + k(x_i - x_o)^{\mathsf{T}}.\end{aligned}$$

and hence

$$\begin{aligned}\frac{\partial V(x)}{\partial x_i}\dot{x}_i =& \left[-\phi_i^{\mathsf{T}} + k(x_i - x_o)^{\mathsf{T}}\right] \\ &\times \left[\phi_i + \psi_i + k(x_o - x_i) + e_i\right] \\ =& -\|\phi_i + k(x_o - x_i) + (\psi_i + e_i)/2\|^2 \\ &+ \|\psi_i + e_i\|^2/4 \\ \leq& \|\psi_i + e_i\|^2/4 \leq (\|\psi_i\|^2 + \|e_i\|^2)/2.\end{aligned}$$

As a result,

$$\frac{dV(x(t))}{dt} = \sum_{i\in\mathbb{N}}\frac{\partial V(x)}{\partial x_i}\dot{x}_i \leq \sum_{i\in\mathbb{N}}(\|\psi_i\|^2 + \|e_i\|^2)/2.$$

For any $t \geq 0$ and any $i \neq j \in \mathbb{N}$, one has

$$\begin{aligned}\int_{\|x_{ij}(t)\|}^{\mu} \alpha(s)ds \leq V(x(t)) &= V(x(0)) + \int_0^t \frac{dV(x(t))}{dt}dt \\ &\leq V(x(0)) + C_1 t + C_2 < \infty\end{aligned}$$

for two finite constants

$$C_1 = N((N-1)\epsilon\delta)^2/2,\ C_2 = \sum_{i\in\mathbb{N}}\int_0^{\infty}\|e_i(t)\|^2 dt/2.$$

Finally, the fact $\int_{\|x_{ij}(t)\|}^{\mu} \alpha(s)ds < \infty$ implies $\|x_{ij}(t)\| > d$ and thus completes the proof.

It suffices to prove the property (P3) if a contradiction is shown from the assumption that the set $\mathcal{S}$ is an invariant set. Under the assumption, the system state $x(0) \in \mathcal{S}$ implies $x(t) \in \mathcal{S}, \forall t \geq 0$, along the closed-loop dynamics. Therefore, all the vehicles and the target are always aligned in a straight line. Due to the proved collision avoidance feature, the sequence of the vehicles along the line does not change. So, one can denote the two end vehicles by $\hbar \neq \ell \in \mathbb{N}$.

Select a constant $\sigma = \min\{k(N-1)d/4, \epsilon\delta/2, k\mu/2\}$. From the proof of (P1), i.e., $\lim_{t\to\infty} \bar{x}(t) - x_o = 0$, there exists T such that x_o is between $x_\hbar(t)$ and $x_\ell(t)$, for all $t \geq T$. Also, $\|e_i(t)\| \leq \sigma,\ i \in \mathbb{N}$, for all $t \geq T$ as $\lim_{t\to\infty} e_i(t) = 0$.

There exists $T_1 \geq T$ such that $\mathcal{N}_\hbar(T_1) \cup \mathcal{N}_\ell(T_1) \neq \emptyset$. Otherwise, the speeds of $\hbar$ and ℓ, towards x_o, are

$$\begin{aligned} u_\hbar \cdot x_{o\hbar}/\|x_{o\hbar}\| &= -kx_{\hbar o} \cdot x_{o\hbar}/\|x_{o\hbar}\| + e_\hbar \cdot x_{o\hbar}/\|x_{o\hbar}\| \\ &= k\|x_{o\hbar}\| + e_\hbar \cdot x_{o\hbar}/\|x_{o\hbar}\| \end{aligned}$$

and

$$u_\ell \cdot x_{o\ell}/\|x_{o\ell}\| = k\|x_{o\ell}\| + e_\ell \cdot x_{o\ell}/\|x_{o\ell}\|,$$

respectively. So, the relative speed between $\hbar$ and ℓ is

$$\begin{aligned} &k\|x_{\hbar\ell}\| + e_\hbar \cdot x_{o\hbar}/\|x_{o\hbar}\| + e_\ell \cdot x_{o\ell}/\|x_{o\ell}\| \\ &\quad > k(N-1)d - 2\sigma \geq k(N-1)d/2. \end{aligned}$$

It contradicts that $\hbar$ and ℓ never collide. Without loss of generality, let $\mathcal{N}_\hbar(T_1) \neq \emptyset$. Denote κ be the nearest neighbor of $\hbar$, i.e., no vehicle exists between $\hbar$ and κ. Next, we consider two cases.

(i) κ is between $\hbar$ and x_o at T_1. [1]

(ii) κ is not between $\hbar$ and x_o at T_1. So, $\|x_{\hbar o}(T_1)\| \leq \|x_{\hbar\kappa}(T_1)\| \leq \mu$. Next, we consider two sub-cases.

(ii-a) κ is always between ℓ and x_o or on x_o for $t \geq T_1$. Then, the speed of ℓ towards x_o is

$$\begin{aligned} u_\ell \cdot x_{o\ell}/\|x_{o\ell}\| &= k\|x_{o\ell}\| + e_\ell \cdot x_{o\ell}/\|x_{o\ell}\| \\ &> k(N-2)d - \sigma \geq k(N-2)d/2. \end{aligned}$$

There exists $T_2 \geq T_1$ such that $\mathcal{N}_\ell(T_2) \neq \emptyset$ that includes a vehicle between ℓ and x_o, for $N > 3$.

(ii-b) κ is not always between ℓ and x_o or on x_o for $t \geq T_1$. So, let $T_3 \geq T_1$ be the first time such that $x_\kappa(T_3) = x_o$ and it moves to $\hbar$ for $t > T_3$. One has $\|x_{\hbar o}(t)\| \leq \mu$ for $t \in [T_1, T_3]$. In fact, for any $\|x_{\hbar o}(t)\| = \mu$, the speed of $\hbar$ towards x_o is

$$\begin{aligned} u_\hbar \cdot x_{o\hbar}/\|x_{o\hbar}\| &= k\|x_{o\hbar}\| + e_\hbar \cdot x_{o\hbar}/\|x_{o\hbar}\| \\ &> k\mu - \sigma \geq k\mu/2. \end{aligned}$$

Therefore, there exists $\tilde{T} > 0$ such that $\mathcal{N}_\hbar(T_3 + \tilde{T}) \neq \emptyset$ and κ is between $\hbar$ and x_o at $T_3 + \tilde{T}$.

From above, one can conclude that at T_1 (or $T_2, T_3 + \tilde{T}$), the end vehicle $\hbar$ (or ℓ, $\hbar$) has a neighbor that is between itself and x_o. For instance, at T_1, $\hbar$ has a neighbor that is between itself and x_o, $\hbar$ has a counter clockwise speed $\geq \epsilon\delta - \sigma \geq \epsilon\delta/2$ around x_o, that is, the angle of the velocity vector is within $(\angle(x_\hbar - x_o), \angle(x_\hbar - x_o) + \pi)$. Let κ' be the vehicle (always exists) that has a neighbor $\hbar$ or a neighbor between itself and $\hbar$ but no neighbor between itself and x_o. Then κ' has a clockwise speed $\geq \epsilon\delta - \sigma \geq \epsilon\delta/2$ around x_o, that is, the angle of the velocity vector is within $(\angle(x_{\kappa'} - x_o) - \pi, \angle(x_{\kappa'} - x_o))$. After the time instant T_1, $\hbar$, κ' and x_o do not stay on a straight line. ∎

[1] We say a vehicle a is between b and c at t if $x_a(t) = \lambda x_b(t) + (1-\lambda)x_c(t)$ for $0 < \lambda < 1$.

Remark 2.4 The theorem shows that the center of the vehicles, $\bar{x}$, converges to the target x_o. Due to the component $k(x_o - x_i)$, each agent x_i never diverges from the target x_o. Also, the center is not on the boundary of the convex hull when it is not a singleton formation. Therefore, the fact that the center converges to the target implies that the target will eventually enter into the convex hull.

Remark 2.5 In the theorem, we prove (P3) for $N > 3$ because a singleton formation may theoretically exist for $N = 3$ for a special initial setting where three vehicles are initially in a singleton formation, one vehicle is overlapped with the target, and the other two are aligned symmetrically about the target. For this special setting, it is easy to see that the rotation function $\sum_{j \in \mathcal{N}_i} \beta(x_i, x_j, x_o) R x_{ij}/\|x_{ij}\|$ in (2) is invalid to break the singleton formation. Nevertheless, this stalemate can be easily broken when the positions of the three vehicles are slightly perturbed.

3 Simulation

The simulation is conducted for five vehicles equipped with the controller $u_i = u_i^o$ defined in (2) for

$$\alpha(s) = \begin{cases} (s-d)^{-1} - (\mu - d)^{-1}, & s \in (d, \mu) \\ 0, & s \in [\mu, \infty) \end{cases},$$

$\mu = 3$, $d = 1$, $\epsilon = 0.05$, $\delta = 10°$, and $k = 1$. The first result, plotted in the top graph of Fig. 1, shows the target-fencing trajectories with the initial positions of the five vehicles arbitrarily selected. In particular, the target at $x_o = [3, 10]^\intercal$ is asymptotically fenced by the convex hull of the five vehicles, as expected by the property (P1). During the evaluation, collision avoidance formulated in the property (P2) is demonstrated in Fig 2 with the curve representing the minimal distance among two adjacent vehicles, i.e., $\min_{i \neq j \in \mathbb{N}} \|x_i(t) - x_j(t)\|$, lower bounded by d. To verify the functionality of the rotation component, the initial vehicle distribution is deliberately set as a singleton formation in the other two graphs of Fig. 1. A proper target-fencing behavior is still achieved in the middle graph, which is consistent with the property (P3) that the singleton formation is not an invariant set. The profile under the controller with $\epsilon = 0$ is shown in the bottom graph where the vehicle trajectories reduce to be within the trivial set of singleton formation.

Theorem 2.1 holds for $N > 3$. The special case with $N = 3$ is discussed in Remark 2.5. For example, the rotation force is invalid for the special initial distribution $[7, 10, 3, 10, -1, 10]^\intercal$, as shown in the top graph of Fig. 3. A slight modification of the initial positions to $[6.5, 10, 3, 10, -1, 10]^\intercal$ makes a proper target-fencing behavior again, as shown in the bottom graph of Fig. 3.

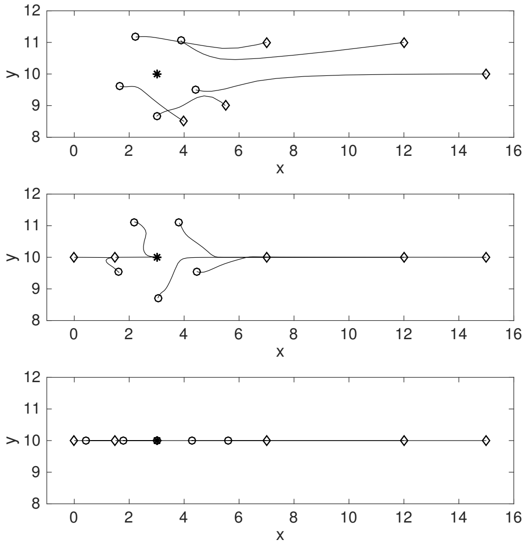

Fig. 1. Target-fencing trajectories of five vehicles moving in a two-dimensional plane ($*$: target position, $\diamond$: initial vehicle position, $\circ$: final vehicle position at the 5th second.)

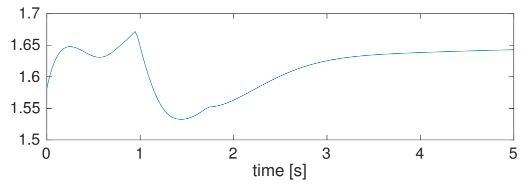

Fig. 2. Minimal distance among two adjacent vehicles.

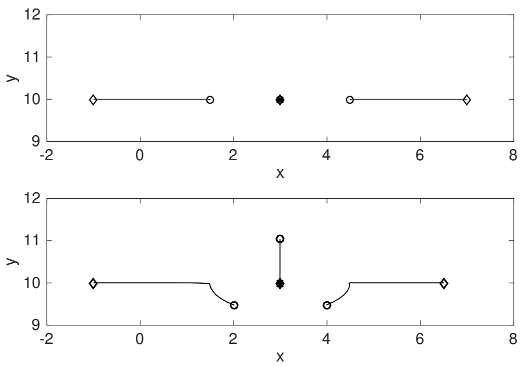

Fig. 3. Target-fencing trajectories of three vehicles moving in a two-dimensional plane.

4 Conclusion

The target-fencing problem has been solved in this paper by a novel autonomous controller equipped with three fundamental functionalities: attraction, repulsion, and rotation. The result holds subject to exponentially vanishing disturbance. The disturbance may also represent the error from an inner velocity regulation controller when vehicle dynamics are considered. It requires further investigation in the future research.

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